Fuzzy Newsboy Vendor Supply Chain Model for Exponentially Distributed Demand

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Abstract:

Market demands of the perishable produce are volatile. Hence, demand of such items is fuzzy random variable. The appealing demand distribution of such item is exponential. The parameter of exponential distribution is obtained. Fuzzy triangular numbers are used to obtain an optimal order quantity. The retailers andmanufacturers profit is obtained under decentralized supply chain. Single period (newsboy) inventory is used to obtain an optimal order quantity. At the end of the day, all the remaining items are rotted. Hence there is no gain from unsold items. Also, some items are defective at the time of purchase.

Keywords: Single period inventory model, Fuzzy random variable, optimal order quantity.

1. Introduction:

Supply chain is the process to transfer the manufactured products from the manufacturer to the customer. Supply chain includes several entities such as manufacturers, suppliers, distributors, retailers, customers etc. If all entities are independent to each other and they are taking their decision independently then that supply chain is called as decentralized supply chain and if all the entities are related to each other and they are taking their decision by one hand then that supply chain is called as centralized supply chain. In a centralized decision making system, there exist a central authority to take the decision, whereas in decentralized system every individual entities can take their own decisions. Both the decision making system have their advantages and disadvantages. The customer is an integral part of supply chain. The main objective of supply chain is to satisfy the customers demand. In practice, it is very difficult to satisfy the customers demand. If the number of order is less than the customers demand then retailer met with loss. If the number of order is greater than the customers demand then retailer will met the surplus inventory and have to sell that surplus items with low price. The supply chain is a real network. Each stage of supply chain is connected through the flow of products. This flow of direction will be in both directions. In traditional supply chain the focus of the integration of SCN is single objective such as minimize the cost or maximize the profit. Supply chain designs how to choose the entities and distribute goods to satisfy the customers demand and minimize the total cost.

Fuzzy set theory is widely used in various fields for different problems which is related to supply chain. Harris et al.(1915) developed the classical economic order quantity model with known constant demand. In the inventory system there exist parameters and constants which are uncertain. When parameters and constants are uncertain then they are usually treated as probability theory. In some cases uncertainties can be defined as fuzziness or vagueness, which are characterized by fuzzy numbers of the fuzzy set. Fuzzy set theory introduced by Zadeh (1965). He also introduced development of fuzzy set by membership function, fuzzy number, triangular fuzzy number, trapezoidal fuzzy number, support of fuzzy set, alpha–cut, convex fuzzy set. Kwakernaak (1978) discussed fuzziness in the context of multivalued logic. Expectation of fuzzy random variable, characteristic function of fuzzy events, probabilities related to fuzzy random variable, conditional expectation and various theorems related to independent fuzzy random variable. Puri et al. (1986) introduced the concept of fuzzy random variable, expectation of fuzzy random variable and definition of expectation generalizes the integral of set-valued function. Petrovic et al. (1996) developed two fuzzy models for the newsboy problem for uncertain environment. He assumed that demand and costs are uncertain and obtained optimal order quantities. H. Chang (2004) investigated the application of fuzzy set theory to the EOQ model with imperfect quality items. He also obtained optimal order lot size to maximize the total profit by using signed distance and ranking method of fuzzy number. A. Eroglu(2007) developed a model to obtain optimal order quantity with defective items and shortages backordered. Dutta et al. (2005) introduced a single period inventory problem in an imprecise and uncertain mixed environment. He obtained optimal order quantity by using a grade mean integration. Arnold F. Shapiro (2009) discussed the important sources of uncertainty, fuzziness and randomness. He also introduced fuzzy random variable and difference between fuzziness and randomness. Zhang et al. (2010) introduced uncertain and fuzzy demand by a fuzzy random variable in a supply chain system based on two-level buyback contract for a newsboy vendor model with single cycle. For numerical calculation, he considered demand as a fuzzy random variable with uniform distribution for single cycle inventory model and he obtained optimal order quantity, retailers profit, suppliers profit and total supply chain profit for decentralized and centralized supply chain system. Dutta et al. (2010) presented an inventory model for single-period with maximizing its expected profit under fuzzy environment. Chen et al. (2011) proposed an analysis method for single-period (newsboy) inventory problem with fuzzy demands and incremental quantity discounts. The proposed method based on ranking fuzzy number and optimization theory. Zhang et al.(2014) introduced uncertain and fuzzy demand by a fuzzy random variable in a supply chain system based on two-level buyback contract for a newsboy vendor model with single cycle. For numerical calculation, he considered demand as a fuzzy random variable with normal distribution for single cycle inventory model and he obtained optimal order quantity, retailers profit, suppliers profit and total supply chain profit for decentralized and centralized supply chain system. The analysis is made for centralized and decentralized system. He showed that centralized system is better than decentralized system.

2. Preliminaries:

Definition: 1. Fuzzy Number: (Zadeh L.A. 1965)

A fuzzy set $\tilde{A}: R \to [0,1]$ is said to be fuzzy number if it satisfies,

- **i)** $\tilde{A}$ is normal.
- **ii)** $\tilde{A}$ is fuzzy convex.
- **iii)** $\tilde{A}$ is upper semicontinuous.
- **iv)** The support of $\tilde{A}$ is compact.

Definition: 2. Membership function: (Zadeh L.A. 1965)

For any set X, a membership function on X is any function from X to real unity interval [0,1]. Membership function on X represent fuzzy subset of X. Membership function of fuzzy set A is denoted by $\mu_A(x)$. For an element of x of X, the value $\mu_A(x)$ is called as membership degree of x in fuzzy set A.

Definition: 3.Support: (Zadeh L.A. 1965)

Let $\tilde{A}$be a fuzzy set of X, the support of $\tilde{A}$ is denoted by *supp(A)*, is a crisp subset of X whose elements all have non-zero membership grades in A

Definition: 4. Normal Fuzzy Set:(Zadeh L.A. 1965)

A fuzzy subset $\tilde{A}$ of a classical set X is called normal fuzzy set if there exist $x \in X$ such that $\mu_A(x) = 1$ otherwise $\tilde{A}$ is subnormal.

Definition: 5. Triangular Fuzzy Number:(Zadeh L.A. 1965)

A triangular fuzzy number$\tilde{A}$ can be expressed as $[l, m, n]$ and its membership function is: $\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ \frac{n-x}{n-m}, & m \le x \le n \end{cases}$

Definition: 6. Trapezoidal Fuzzy Number:(Zadeh L.A. 1965)

A trapezoidal fuzzy number$\tilde{A}$ can be expressed as $[k, l, m, n]$ and its membership function is:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-k}{l-k}, if\ k \le x \le l, \\ 1, \quad if\ l \le x \le m, \\ \frac{n-x}{n-m}, if\ m \le x \le n \end{cases}$$

Definition: 7. alpha-cut (Kwakernaak 1978)

Let $\tilde{A}$ be a fuzzy set defined on universal set $X$and any number α, α∈ [0,1]. The α-cut is denoted by A_α and is given by:

$$A_{\boldsymbol{\alpha}} = \{x/A(x) \ge \alpha\}$$

Alpha cut contains all the elements of the universal set X whose membership grade A are greater than or equal to the specified valueα.

Definition: 8. Fuzzy Random Variable (Kwakernaak 1978)

Let $(\Re, \mu, \wp)$ be the probability space and X be a random variable on $(\Re, \mu, \wp)$ with a probability density function $f(x)$. Fuzzy random variable $\tilde{X}$is mapping from R to a family of fuzzy numbers, i.e. $\tilde{X}: x \to \tilde{X}_{(x)} \in F$,where F denotes fuzzy set. Fuzzy random variables are the random variables that are valued fuzzy numbers.

For given $\alpha \in (0,1]$, suppose that the $\alpha -$ cut$\tilde{X}_{(x)_\alpha} = [\tilde{X}_{(x)_{\alpha^-}}, \tilde{X}_{(x)_{\alpha^+}}]$

Let $[\tilde{X}_{(x)_{\alpha^-}}, \tilde{X}_{(x)_{\alpha^+}}]$ denote left end point and right end point of the $\alpha -$ cut $\tilde{X}_{(x)_\alpha}$

of $\tilde{X}_{(x)}$, where $\tilde{X}_{(x)_{\alpha^-}}$, $\tilde{X}_{(x)_{\alpha^+}}$ are real valued random variable.

Definition: 9. Fuzzy Expectation (Kwakernaak 1978)

The fuzzy expectation of fuzzy random variable $\tilde{X}$ is defined as:

$$E(\widetilde{X}) = \bigcup_{\alpha\in[0,1]} \alpha[\int_{-\infty}^{\infty} \tilde{X}^L\alpha\, dp, \int_{-\infty}^{\infty} \tilde{X}^R\alpha\, dp]$$

Definition: 10.Signed Distance of Fuzzy Expectation (Yao and Wu, 2000)

Let $\tilde{A}$ be a fuzzy number with alpha-cut $\tilde{A}_\alpha = [\tilde{A}^L\alpha, \tilde{A}^R\alpha]$ then signed distance of fuzzy number $\tilde{A}$ is $(\tilde{A}, 0) = \frac{1}{2}\int_0^1(\tilde{A}^L\alpha + \tilde{A}^R\alpha)\, \mathrm{d}\, \alpha$. Then the signed distance of fuzzy expectation $E(\tilde{X})$is $d\big(E(\tilde{X}), 0\big) = \frac{1}{2}\int_0^1[[E(\tilde{X}^L\alpha)] + \int_0^1[E(\tilde{X}^R\alpha)]d\alpha$

Theorem of Integration by part:

Suppose F and$G$ are differentiable function on $[a, b]F' = f\ and\ G' = g \in R$ then

$$\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$$

Leibnitz Theorem:

$$\frac{d}{dt}[\int_{a(t)}^{b(t)} f(x,t)dx] = \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t} dx + f(b(t),t)b'(t) - f(a(t),t)a'(t)$$

3. Mathematical Model:

3.1 Assumptions:

- **i)** Demand is assumed to be fuzzy random variable.
- **ii)** Demand follows exponential distributed.
- **iii)** Order is placed only once.
- **iv)** Supplier's buyback price is more than the salvage value.
- **v)** Supply chain model consider only one supplier and one retailer.
- **vi)** Life time of item is very short.

Notations:

Following are the notations are used to formulate the mathematical model.

$\tilde{X}$ $-$ Market demands in the sales season a continuous fuzzy random variable.

p $-$Retailer's sale price in the sales season determined by market competition.

w −Suppliers wholesale price.
c −Supplier's production cost.
b −Supplier's buyback price.
s −Reduced price for unsold products, which is same for the retailer and supplier.
g_r −Retailer's shortage cost when order quantity cannot meet the market demand.
g_s −Supplier's shortage cost when order quantity cannot meet the market demand.
λ −Supplier's revenue proportion of the supply chain.
Q −Retailers order quantity.
$T(Q)$ −Overall profit for the supply chain under buyback decentralized decision.
$E(X)$ −Expectation for X.
Q^* −Optimal order quantity.
$\widetilde{RP(Q)}$ −Retailers profit under buyback decentralized decision making.
$\widetilde{SP(Q)}$Retailers profit under buyback decentralized decision making.

3.2 Mathematical Model based on Exponential Random Variable:

Suppose continuous random variable X follows exponential distribution with parameter θ then the probability density function and distribution function of X are:

$$f(x) = \theta e^{-\theta x} \quad , x \geq 0$$

And distribution function is

$$F(x) = 1 - e^{-\theta x}$$

Suppose $\tilde{X}$ is the stochastic demand with p.d.f $f(x)$. It is very difficult to obtain correct value of $X(x)$ corresponding to the best real stochastic demand Xdue to incomplete information about complex supply-demand system. To obtain the best value of $X(x)$ can be obtained by rough interval,

$$\widetilde{X(x)} = \left(X(x) - \delta_{1,} X(x), X(x) + \delta_2\right)$$

Where, $\delta_{1,}$ and δ_2 are the reflect decision- makers of fuzzy perception of $X(x)$

.

Following are the some conditions for buyback contract:

$$p > w > b > 0, \; w > c > s \geq 0, \; p > g_r \geq 0, w > g_s \geq 0$$

When $b > s \geq 0$

holds the supplier's buyback price is greater than the salvage value.

Decentralized Decision Making Under a Buyback Contract:

Every supply chain includes several entities and they try to improve the performance independently and enhance the performance of supply chain. In this situation, retailers profit can be evaluated as (Zhang et al.2014).

$$\widetilde{RP(Q)} = p\,min\left(Q, \tilde{X}\right) - wQ + b(Q - \tilde{X})^+ - g_r(\tilde{X} - Q)^+ \qquad (1)$$

$$\widetilde{SP(w,b)} = (s - b)(Q - \tilde{X})^+ - g_s(\tilde{X} - Q)^+ + (w - c)Q \qquad (2)$$

Where,

$$(Q - \tilde{X})^+ = \max\left(Q - \tilde{X}, 0\right), \quad (\tilde{X} - Q)^+ = \max(\tilde{X} - Q, 0)$$

$$\min(Q, \tilde{X}) = \tilde{X} - (\tilde{X} - Q)^+ , (Q - \tilde{X})^+ = Q - \tilde{X} + (\tilde{X} - Q)^+ \qquad (3)$$

Using equation (3) in equation (1) and (2), the retailers and suppliers profit becomes:

$$\widetilde{RP(Q)} = (p - b)\tilde{X} + (b - g_r - p)(\tilde{X} - Q)^+ + (b - w)Q$$

$$\widetilde{SP(w,b)} = (s-b-g_s)(\tilde{X}-Q)^+ + (s-b-w-c)Q - (s-b)\tilde{X}$$

Total supply chain profit is the sum of retailers and suppliers profit:

$$T(Q) = \widetilde{RP(Q)} + \widetilde{SP(w,b)}$$

$$T(Q) = (p-s)\tilde{X} + (s-g_r-g_s-w-c)(\tilde{X}-Q)^+ + (s-c)Q$$

The retailers expected profit is;

$$\mathrm{E}[\widetilde{RP(Q)}] = (p-b)E(\tilde{X}) + (b-g_r-p)E(\tilde{X}-Q)^+ + (b-w)Q \qquad (4)$$

The expected overall profit for the supply chain is;

$$E[\widetilde{SP(w,b)}] = (p-s)E(\tilde{X}) + (s-g_r-g_s-w-c)E(\tilde{X}-Q)^+ + (s-c)Q \qquad (5)$$

For any given Q, a stock value of X for a demand $X \geq Q$ indicates the occurrence of a stock out. The value of continuous fuzzy random demand lies in the fuzzy interval.

$$[x-\delta_1,\ x,\ x+\delta_2],$$

Suppose $Y = X - Q$,as X is a random variable the Y should also be a random variable that is linearly correlated to X. To obtain the expression for expected shortage quantity we use

$\widetilde{Y(x)} = (\tilde{X}-Q)^+$ is a continuous fuzzy random variable.

The demand are expressed in terms of triangular fuzzy numbers $(\tilde{X}-\delta_1, \tilde{X}, \tilde{X}+\delta_2)$

The shortages can also be expressed as triangular fuzzy numbers

$$[(\tilde{X}-Q)^+ - \delta_1, (\tilde{X}-Q)^+, (\tilde{X}-Q)^+ + \delta_2]$$

Using α-cuts, the triangular fuzzy number can be expressed as:

$$[(\tilde{X}-Q)^+ - \delta_1 + \alpha\delta_1, (\tilde{X}-Q)^+, (\tilde{X}-Q)^+ + \delta_2 - \alpha\delta_2]$$

The retailer’s fuzzy expected profit is

$$\begin{aligned} M[E[\check{R}P(Q)]] &= (p-b)\left[E(X) + \frac{\delta_2-\delta_1}{6}\right] + (b-w)Q \\ &+ (b-g_r-p)\left[\int_0^1 \alpha[[\int_{Q+\delta_1-\alpha\delta_1}^{\infty} \mathrm{xf(x)}\,dx + (-\delta_1+\alpha\delta_1-Q)(1-F(Q+\delta_1-\alpha\delta_1)]d\alpha\right. \\ &+ \int_0^1 \alpha[\int_{Q+\alpha\delta_2-\delta_2}^{\infty} \mathrm{x\,f(x)dx} + (\delta_2-\alpha\delta_2-\mathrm{Q})(1-\mathrm{F}(\mathrm{Q}+\alpha\delta_2-\delta_2\Big))]d\alpha] \end{aligned}$$

After evaluating the above equation, the simplified form of retailer’s fuzzy expected profit is:

$$\begin{aligned} M[E[\check{R}P(Q)]] &= (p-b)\left[E(X) + \frac{\delta_2-\delta_1}{6}\right] + (b-w)Q + (b-g_r-p)[(\frac{1}{\theta}+1)e^{-\theta(Q+\delta 1)}\{\left(\frac{e^{\theta\delta 1}}{\theta\delta 1}\right) - \frac{1}{(\theta\delta 1)^2}(e^{\theta\delta 1-1)}\} \\ &+ [(\frac{1}{\theta}+1)e^{-\theta(Q-\delta 2)}\{\left(\frac{-e^{\theta\delta 2}}{\theta\delta 2}\right) - \frac{1}{(\theta\delta 2)^2}(e^{-\theta\delta 2-1)}\}] \end{aligned}$$

(6)

Differentiate equation (6) with respect to Q and using Leibnitz Theorem and equating to zero we get equation (7)

$$\int_0^1 \alpha[F(Q+\delta_1-\alpha\delta_1)+F(Q+\alpha\delta_2-\delta_2)]d\alpha = \frac{p+g_r-w}{p+g_r-b} \qquad (7)$$

We can find the value of Q by assuming exponential distribution. After simplification and assuming exponential distribution, the value of Q is:

$$Q=\frac{1}{\theta}\left\{\log(p+g_r-b)-\log(w-b)+\log\left\{\left(\frac{1}{\theta\delta_1}-\frac{(1-e^{-\theta\delta_1})}{(\theta\delta_1)^2}\right)-\left(\frac{1}{\theta\delta_2}+\frac{(1-e^{\theta\delta_2})}{(\theta\delta_2)^2}\right)\right\}\right\} \qquad (8)$$

If Q is the optimal order quantity the manufacturers profit in decentralized system is:

$$M(Q)=(w-c)Q \qquad (9)$$

4. Results and Discussion:-

The expected profit for retailer, supplier and total supply chain are calculated and the results are mentioned in Table -1. To clarify the decision making process we set the parameters $\lambda = 0.4, \delta1 = 400, \delta2 = 100, p = 437, w = 374.5714,$ repurchase price $= 318.3857$. Shortage costs for both retailer and suppliers are very low, then $g_r = g_s = 0$

Then optimal order quantity Q^* which can be calculated by equation (8) which gives $Q^* = 294.3935$. The expected profit for the retailer can be obtained from equation (6)

Which gives $T(Q) = 291584.9$.

As the value of λ increases then order quantity Q and manufacturers profit decreases and retailers profit increases. As the λ decreases manufacturers profit increases and retailers profit and order quantity increases. The suppliers expected profit and retailers expected profit decreases by increasing the value of δ_1 The suppliers expected profit and retailers profit increases by increasing the value of δ_2

Table-1. Sensitivity Analysis of Optimal order Quantity, Retailers Profit, Manufacturers Profit

λ	$\delta1$	$\delta2$	b	c	w	Q	$M(Q)$	$\check{R}P(Q)$	$T(Q)$
0.4	400	100	318.3857	280.9286	374.5714	294.3935	27567.84636	264017.1	291584.9
0.3	400	100	350.3048	296.0323	394.7097	312.8417	30870.41339	222956.6	253827
0.2	400	100	380.4471	308.4706	411.2941	357.6569	36775.5432	184539.2	221314.7
0.1	400	100	409.2446	318.8919	425.1892	466.3819	43673.3303	148700.7	198275.8
0.4	300	200	318.3857	280.9286	374.5714	327.6321	32329.89198	300908.1	344581.4
0.3	300	200	350.3048	296.0323	394.7097	346.0803	35585.19638	254093.4	286423.3
0.2	300	200	380.4471	308.4706	411.2941	390.8956	41551.14686	210323.2	245908.4
0.1	300	200	409.2446	318.8919	425.1892	499.6205	46785.88754	169435.3	210986.4
0.4	200	300	318.3857	280.9286	374.5714	363.3274	35852.21226	463903.4	510689.3
0.3	200	300	350.3048	296.0323	394.7097	381.7756	37672.63314	392931.3	430603.9
0.2	200	300	380.4471	308.4706	411.2941	426.5909	43863.58007	326435.4	370299
0.1	200	300	409.2446	318.8919	425.1892	535.3158	56902.62419	263905.2	320807.8
0.4	100	400	318.3857	280.9286	374.5714	401.5995	37606.92174	784139.1	821746
0.3	100	400	350.3048	296.0323	394.7097	420.0477	41449.22541	666060.8	707510
0.2	100	400	380.4471	308.4706	411.2941	464.863	47798.8523	555174	602972.9
0.1	100	400	409.2446	318.8919	425.1892	573.5879	60970.84508	450307.9	511278.7

Note- Maximum optimal order quantity Q=573.5879 for $\delta_{1=100,}$ $\delta_{2=400}$ and α=0.1

5. Conclusion:

Here, we considered the newsboy vendor problem in which demand follows exponential distribution and which is fuzzy. We considered a two level supply chain model based on buyback contract under fuzzy random demand. We made the analysis for decentralized system by fixing related parameters. The expected profit for retailer and supplier decrease in the supply chain. As the value of δ_2 increases then the retailer order quantity increases but the intrest rate decreases for both the parties. Hence, suppliers need to adopt relevant policies to encourage the retailers to plan the order.

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