SOLUTION TO EXTENDED TRANSPORTATION PROBLEM BASED ON COMMON SET OF WEIGHTS

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Abstract: The main objective of transportation problem is the shipping of homogeneous goods from various origins to various destinations. In classical transportation problem, the distribution of goods is made by considering only one quantitative characteristic per shipment link.However,in real life, it is observed that the decision about the distribution of goods is based on several factors. In this paper, the transportation problem is extended by considering multiple incommensurate inputs and outputs for each shipment link, known as extended transportation problem (ETP). Common set of weights approach is proposed to compute the relative efficiency of each shipment link. The shipment plan with maximum efficiency is considered as optimal solution to ETP. A numerical example is discussed in order to validate the applicability of proposed approach.

Keywords: Data Envelopment Analysis (DEA); Decision Making Unit(DMU); Relative Efficiency; Extended Transportation Problem (ETP).

INTRODUCTION

The transportation problem is a subclass of linear programming problem. It has the great importance in all type of production industries. The main objective of transportation problem is the shipping of homogeneous commodity from various origins to various destinations such that total cost of transportation will be minimum. The formulation of classical transportation problem is based on only cost or profit for each possible shipment link. But it is observed that in many real situations the shipment of goods is made by considering several kinds of variables such as cost, distance, shipment value, manpower, profit, mode of transportation,etc. These variables are classified as input and output variables and should be incorporated for each possible shipment link. The decision makers may have different aims to achieve for each possible shipment link, which may conflict to each other. In such situation, an optimal transportation plan is decided by taking into account the relative efficiency of each shipment link.

REVIEW OF LITERATURE

The distribution of homogeneous production from several sources to numerous localities was originally mentioned as transportation problem by Hitchcock (1941). Data Envelopment Analysis (DEA) is a mathematical approach to compute relative efficiency of decision making units (DMUs). The comparative efficiency of a set of DMUs such as airlines, railways, banks, automobile manufacturers, hospitals, universities, municipal-corporations, educational institutes, etc. can be obtained by using DEA. Charnes, Cooper, and Rhodes (1978) introduced DEA technique in the literature. A number of DEA applications and research have led to many new developments in concepts and methodologies related to the DEA efficiency analysis. Charnes et.al.(1978) suggested a model to compute relative efficiency of various DMUs, named as CCR model.The literature available on ETP is very limited. Chen and Lu (2007) extended the assignment problem by considering multiple inputs and outputs. Alireza Amirteimoori (2012) has applied the idea of Chen and Lu to transportation problem using CCR model.

DEA MODELS

Let us assume that there be n DMUs, each consumes varying amounts of m-different inputs to produce s- different outputs. Let $y_{rj}; r=1,2,3,...s$ and $x_{ij}; i=1,2,3,...m$ denotes the non-negative input and output values respectively for j[th] DMU denoted as DMU_j ; j =1,2,....n. A particular DMU from all possible DMUs is considered for evaluation, denoted as DMU_o. This DMU_o is placed in the functional form to maximize output and also taken in the constraints. Charnes et.al.(1978) suggested input oriented fractional programming model (CCR model) to compute relative efficiency of various DMUs as

$$\text{Maximize} \left\{ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \right\}$$

Subject to:

$$\left\{ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \right\} \leq 1 \qquad \forall_j$$

$$u_r, v_i \geq \varepsilon, \qquad \forall_{r,i}$$

$$\varepsilon > 0. \qquad (1)$$

Similarly, CCR (output oriented) model can be written in the form of fractional programming model as:

$$\text{Minimize} \left\{ \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{r=1}^{s} u_r y_{ro}} \right\}$$

Subject to:

$$\left\{ \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{r=1}^{s} u_r y_{ro}} \right\} \geq 1 \qquad \forall_j$$

$$u_r, v_i \geq \varepsilon, \qquad \forall_{r,i}$$

$$\varepsilon > 0. \qquad (2)$$

Where, $u_r; r = 1,2,3,...s$ and $v_i; i = 1,2,3,...m$ are the weights associated with rth output and ith input variable respectively and ε is a non-Archimedean infinitesimal.

MULTI-OBJECTIVE LINEAR PROGRAMMING

Multi-objective linear programming problem (MOLP) helps to Optimize several linear objective functions over a set of linear constraints. Boychuk and Ovchinnikov(1973) have suggested the following method to solve MOLP.

Let $f_i(x)$; i=1,2,…,m and $g_j(x)$; j=1,2,…,n are objective functions and X is a feasible region. Let $f_i^*(x)$ = max $f_i(x)$; i=1,2,…,m and $g_j^*(x)$ = min $g_j(x)$; j=1,2,…,n such that x ε X. Then, the solution which satisfies both objective functions can be obtained by solving linear programming problem as:

$$\text{Max}\left\{\sum_{i=1}^{m}\frac{f_i(x)}{f_i^*}-\sum_{j=1}^{n}\frac{g_j(x)}{g_j^*}\right\}$$

such that x ε X.

PROPOSED METHOD TO SOLVE ETP

Three stage method is proposed to solve ETP. In stage I, separate weights for inputs and outputs of various shipment links (DMU_s) are decided by solving CCR models. The set of common weights are decided by MOLP in stage II and the optimal shipment plan with maximum efficiency is obtained in stage III.

Stage I

The multiplier (weights) u_r to output variable $y_{ij}^{(r)}$ and v_k to input variable $x_{ij}^{(k)}$ for each shipment link (i, j) are determined by using the CCR (input oriented) model. Charnes and Cooper (1962) transformation is used to write the CCR models (input oriented) in the form of linear programming model as:

$$e_{ij}^{(1)} = \text{Max}\left\{\sum_{r=1}^{t}u_r y_{ij}^{(r)}\right\}$$

Subject to:

$$\sum_{k=1}^{s}v_k x_{ij}^{(k)} = 1$$

$$\sum_{r=1}^{t}u_r y_{ij}^{(r)} - \sum_{k=1}^{s}v_k x_{ij}^{(k)} \le 0 \quad \forall i = 1,2,...,m\ ;\ j = 1,2,...,n$$

$$u_r \ge \varepsilon \qquad ;r = 1,2,.....t$$

$$v_k \ge \varepsilon \qquad ;k = 1,2,....s$$

$$\varepsilon > 0. \qquad (3)$$

These relative efficiencies are obtained by fixing origin and changing the destinations to a particular shipment link.

Similarly, the other set of multiplier (weights) u'_r to output variable $y_{ij}^{(r)}$ and v'_k to input variable $x_{ij}^{(k)}$ for each shipment link (i, j) are determined by using the CCR (output oriented) model as:

$$e_{ij}^{(2)} = \text{Min } \left\{ \sum_{k=1}^{s} v'_k x_{ij}^{(k)} \right\}$$

Subject to:

$$\sum_{r=1}^{t} u'_r y_{ij}^{(r)} = 1$$

$$\sum_{r=1}^{t} u'_r y_{ij}^{(r)} - \sum_{k=1}^{s} v'_k x_{ij}^{(k)} \le 0 \ \forall i = 1,2,\dots,m \ ; j = 1,2,\dots,n$$

$$u'_r \ge \varepsilon \qquad ;r = 1,2,\dots\dots t$$

$$v'_k \ge \varepsilon \qquad ;k = 1,2,\dots.s$$

$$\varepsilon > 0 \qquad (4)$$

We decide the optimal value for objective function in (3) and (4) with corresponding multipliers to input and to output variables for each shipment link (i, j).

Stage II:

Separate weights to each DMU are decided in stage I. The efficacy of individual DMU with others should be compare with some common base. This is achieved by calculating the common set of weights (CSW) using MOLP:

$$Maximize \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \{ \sum_{r=1}^{t} \frac{\bar{u}_r y_{ij}^{(r)}}{e_{ij}^{(1)}} \} - \sum_{i=1}^{m} \sum_{j=1}^{n} \{ \sum_{k=1}^{s} \frac{\bar{v}_k x_{ij}^{(k)}}{e_{ij}^{(2)}} \} \right\}$$

Such that,

$$\sum_{r=1}^{t} \bar{u}_r + \sum_{k=1}^{s} \bar{v}_k = 1$$

$$\sum_{r=1}^{t} \bar{u}_r y_{ij}^{(r)} - \sum_{k=1}^{s} \bar{v}_k x_{ij}^{(k)} \le 0 \ ; i = 1,2,\dots,m; \ j = 1,2,\dots,n$$

$$\bar{u}_r \ge \varepsilon \qquad ;r = 1,2,\dots\dots t$$

$$\bar{v}_k \ge \varepsilon \qquad ;k = 1,2,\dots.s$$

$$\varepsilon > 0 \qquad (5)$$

Suppose $(\bar{u}_r^*, \bar{v}_k^*)$ is the solution of model (5), we find the relative efficiency of (i, j)th shipment link with CSW $(\bar{u}_r^*, \bar{v}_k^*)$ as:

$$e_{ij} = \frac{\sum_{r=1}^{t} \bar{u}_r^* y_{ij}^{(r)}}{\sum_{k=1}^{s} \bar{v}_k^* x_{ij}^{(k)}} \qquad ; i = 1,2,\dots,m \ ; j = 1,2,\dots,n. \qquad (6)$$

Stage III:

Our aim is to make the allocation on the basis of efficient shipment link, which is achieved by solving the model:

$$\textit{Minimise} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}(1-e_{ij})t_{ij}$$

Subject to:

$$\sum_{j=1}^{n} t_{ij} = a_i \qquad \forall_i ,$$

$$\sum_{i=1}^{m} t_{ij} = b_j \qquad \forall_j ,$$

$$t_{ij} \geq 0 \qquad \forall_{i,j} . \qquad (7)$$

NUMERICAL ILLUSTRATION

Suppose a factory produces cement for construction work and marketed with packing of 50 kg. per bag. Suppose the production is shifted to three warehouses as A, B and C. Cement bags are supplied to four wholesalers such as D, E, F and G as per their demands from these warehouses. Triplets in the Table 1, shows per unit cost of transportation, lot size/order size and profit per unit. The outer column shows the capacities at different warehouses and outer raw indicates demands at various dealers.

In this example, per unit cost of transportation and lot size is considered as input variables and per unit profit as output variable. So, the problem is treated as ETP and can be solved by proposed approach. The relative efficiency of each shipment link with weights for input and output variables are obtained by solving equations (3) and given in Table 2. Another set of weights are obtained by solving the equations (4) and tabulated in Table 3. The CSW for wholesaler by fixing warehouse is obtained by solving equation (5). Efficiencies for each shipment link based on CSW are calculated by using equation (6). These calculations are stated in Table 4. The optimal solution to ETP with maximum efficiency is obtained by solving equation (7) and it is as:

t_{AG}=15; t_{BD}=5; t_{BE}=5; t_{CE}=5; t_{CF}=15; t_{CG}=5 with the value of objective function in equation (7) is 6.1165.

CONCLUSION

Proposed method is applicable to the transportation problem involving more than one variables per shipment link. Each shipment link in transportation problem with multiple inputs and multiple outputs is consideredas decision making unit.The efficacy of these DMUs are decided by using DEA technique. Separate weights for input and output variables related with each DMU are obtained by using CCR model. Multi-objective linear programming approach is used to find CSW in order to maximise output by minimising the input at each DMU. It helps to compare efficacy of all shipment links with common base.Three stage method is suggested to obtain the optimal shipment plan with maximum efficiency.The applicability of proposed approach is discussed by an illustrative example.

Table 1: Transportation problem with multiple inputs and outputs per shipment links.

Warehouse	Wholesalers				Capacity (‘ ton)
	D	E	F	G	
A	(2,50,15)	(3,50,18)	(1,100,12)	(2,150,16)	15
B	(1,100,10)	(2,50,20)	(3,100,14)	(4,50,8)	10
C	(3,150,8)	(1,200,12)	(2,50,10)	(4,100,10)	25
Demand (‘ton)	5	10	15	20	

Table 2:The relative efficiency of each shipment link with weights for input and output variables

DMUs	$e_{ij}^{(1)}$	u_1	v_1	v_2
(1,1)	1	.0667	.2	.012
(1,2)	1	.0556	.0001	.02
(1,3)	1	.0833	.5	.005
(1,4)	.7619	.0476	.2857	.0029
(2,1)	1	.1	1.0	.0001
(2,2)	1	.05	.0001	.02
(2,3)	.4667	.0333	.3333	.0001
(2,4)	.4	.05	.0001	.02
(3,1)	.4444	.05556	.2222	.0022
(3,2)	1	.0833	.3333	.0033
(3,3)	1	.10	.0001	.02
(3,4)	.5	.05	.0001	.01

Table 3: The set of weights which minimises weighted input

DMUs	$e_{ij}^{(2)}$	u'_1	v'_1	v'_2
(1,1)	1	.0667	.20	.012
(1,2)	1	.0556	.0001	.02
(1,3)	1	.0833	.5	.005
(1,4)	1.3125	.0625	.375	.0038
(2,1)	1.0008	.10	.9998	.0001
(2,2)	1	.05	.0001	.02
(2,3)	2.1431	.0714	.7140	.0001
(2,4)	2.5	.13	.0001	.05
(3,1)	2.25	.1250	.50	.005
(3,2)	1	.0833	.3333	.0033
(3,3)	1	.10	.0001	.02

(3,4)	2	.10	.0001	.02

Table 4: CSW and Efficiencies for each shipment link based on CSW(e_{ij})

Warehouse	CSW per warehouse			Efficiencies per wholesalers			
	$\bar{u}_1^*$	$\bar{v}_1^*$	$\bar{v}_2^*$	D	E	F	G
A	.1416	.8498	.0085	1	.8571	.9998	.7618
B	.0911	.9087	.0001	.9919	1	.4663	.2003
C	.1984	.7937	.0079	.4444	.9995	1	.4999

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