Solving Linear System of Equations with Various Examples by Using GAUSS Method

Dr. YOGEESH N
Assistant professor in Mathematics
Government First Grade College
BH Road, Tumkur-572102
Government of Karnataka, INDIA
E-Mail: yogeesh.r@gmail.com

ABSTRACT

This paper shows how to find the set of all solutions by solving the system of equations. An algorithm always works called *Gauss' method*. It transforms the system, systematically, into one with a form can is easily solved. Here there are many examples of solving linear systems by Gauss' method. It is fast, easy and safe method. It never loses solutions or picks up extraneous solutions. In addition, there is an idea to write Computational program or C-program of Gaussian elimination to solve linear equations along with explanation.

KEYWORDS

Gauss method, Upper Triangular Form (Forward Sweep), Backward Sweep, Equation Normalization, Partial Pivoting.

INTRODUCTION

A system of linear equations is a group of two or more linear equations with same set of variables.

Definition: A linear equation in variables $x_1, x_2 \ldots., x_n$ has the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \ldots. + a_nx_n = d$$

Where the numbers $a_1 \ldots. a_n \in R$ are the equation's co-efficient and d Є R is the constant. An n-tuple $(s1, s2 \ldots, sn) \in R^n$ is a solution of, or satisfies, that equation if substituting the numbers $s1, s2, .. ., sn$ for the variables gives a true statement: $a_1s1 + a_2s2 + \ldots. + a_nsn = d$.

A system of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots. + a_{1n}x_n &= d_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots. + a_{2n}x_n &= d_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots. + a_{mn}x_n &= d_m \end{aligned}$$

has the solution $(s1, s2 \ldots., sn)$ if that n-tuple is a solution of all of the equations in the system.

Example-1. To solve this system

$$\begin{aligned} 3x_3 &= 9 \\ x_3 + 5x_2 - 2x_3 &= 2 \\ \frac{1}{3}x_1 + 2x_2 &= 3 \end{aligned}$$

We repeatedly transform it until it is in a form that is easy to solve.

Swap row 1 with row 3

$$\begin{aligned} \tfrac{1}{3}x_1 + 2x_2 &= 3 \\ x_1 + 5x_2 - 2x_3 &= 2 \\ 3x_3 &= 9 \end{aligned}$$

Multiply row 1 by 3

$$\begin{aligned} x_1 + 6x_2 &= 9 \\ x_1 + 5x_2 - 2x_3 &= 2 \\ 3x_3 &= 9 \end{aligned}$$

Add – 1 times row 1 to row 2

$$\begin{aligned} x_1 + 6x_2 &= 9 \\ -x_2 - 2x_3 &= -7 \\ 3x_3 &= 9 \end{aligned}$$

The third step is the only nontrivial one. We have mentally multiplied both sides of the first row by -1, mentally added that to the old second row, and written the result in as the new second row.

Now we can find the value of each variable. The bottom equation shows that $x_3 = 3$. Substituting 3 for x_3 in the middle equation shows that $x_2 = 1$. Substituting those two into the top equation gives that $x_1 = 3$ and so the system has a unique solution: the solution set is {(3,1,3)}.

Most of this subsection and the next one consists of examples of solving linear systems by Gauss' method. It is fast and easy. However, before we get to those examples, we will first show that this method is also safe in that it never loses solutions or picks up extraneous solutions.

Theorem (Gauss' method) If a linear system is changed to another by one of these operations

1. An equation is swapped with another
2. An equation has both sides multiplied by a nonzero constant
3. An equation is replaced by the sum of itself and a multiple of another

Then the two systems have the same set of solutions.

Each of those three operations has a restriction. Multiplying a row by 0 is not allowed because obviously that can change the solution set of the system. Similarly, adding a multiple of a row to itself is not allowed because adding – 1 times the row to itself has the effect of multiplying the row by 0. Finally, swapping a row with itself is disallowed to make some results (and besides, self-swapping doesn't accomplish anything).

PROOF. We will cover the equation swap operation here

Consider this swap of row i with row j.

$$\begin{array}{ll} a_1,1x_1 + a_1,2x_2 + \ldots.+a_1,nx_n = d_1 & a_1,1x_1 + a_1,2x_2 + \ldots.+a_1,nx_n = d_1 \\ \vdots & \vdots \\ a_i,1x_1 + a_i,2x_2 + \ldots.+a_i,nx_n = d_i & a_j,1x_1 + a_j,2x_2 + \ldots.+a_j,nx_n = d_j \\ \vdots & \vdots \\ a_j,1x_1 + a_j,2x_2 + \ldots.+a_j,nx_n = d_j & a_i,1x_1 + a_i,2x_2 + \ldots.+a_i,nx_n = d_i \\ \vdots & \vdots \\ a_m,1x_1 + a_m,2x_2 + \ldots.+a_m,nx_n = d_m & a_m,1x_1 + a_m,2x_2 + \ldots.+a_m,nx_n = d_m \end{array}$$

The n-tuple $(s1,\ldots.sn)$ satisfies the system before the swap if and only if substituting the values, the s's, for the variables, the x's, gives true statements: $a_1,1x_1 + a_1,2x_2 + \ldots.+a_1,nx_n = d_1$ and $a_i,1x_1 + a_i,2x_2 + \ldots.+a_i,nx_n = d_i$ and $\ldots a_j,1x_1 + a_j,2x_2 + \ldots.+a_j,nx_n = d_j$ and $\ldots$ $a_m,1x_1 + a_m,2x_2 + \ldots.+a_m,nx_n = d_m$this is exactly the requirement that $(s1,\ldots.sn)$ solves the system after the row swap.

Definition: The three operations from the above Theorem are the elementary reduction operations, or row operations, or Gaussian operations. They are swapping, multiplying by a scalar or rescaling and pivoting.

When writing out the calculations, we will abbreviate 'row i' by 'R_i'. For instance, we will denote a pivot operation by kR_i+R_j, with the row that is changed written second. We will also, to save writing, often list pivot steps together when they use the same R_i.

Example-2. A typical use of Gauss' method is to solve this system.

$$\begin{aligned} x + y &= 0 \\ 2x - y + 3z &= 3 \\ x - 2y - z &= 3 \end{aligned}$$

The first transformation of the system involves using the first row to eliminate the x in the second row and the x in the third. To get rid of the second row's 2x, we multiply the entire first row by -2, add that to the second row, and write the result in as the new second row. To get rid of the third row's x, we multiply the first row by -1, add that to the third row, and write the result in as the new third row.

$$\begin{array}{ll} & x + y = 0 \\ -R_1 + R_3 & -3y + 3z = 3 \\ -2R_1 + R_2 & -3y - z = 3 \end{array}$$

(Note that the two R_1 steps $-2R_1 + R_2$ and $-R_1 + R_3$ are written as one operation).

In this second system, the last two equations involve only two unknowns.

To finish we transform the second system into a third system, where the last equation involves only one unknown. This transformation uses the second row to eliminate y from the third row.

$$\begin{array}{ll} & x + y = 0 \\ -R_2 + R_3 & -3y + 3z = 3 \\ & -4z = 3 \end{array}$$

Now we are set up for the solution. The third row shows that, $z = 0$. Substitute that back into the second row to get y = -1 and then substitute back into the first row to get $x = 1$.

Example-3. The reduction

$$\begin{array}{lll} x + y + z = 9 & & x + y + z = 9 \\ 2x + 4y - 3z = 1 & -2R_1 + R_2 & 2y - 5z = -17 \\ 3x + 6y - 5z = 0 & -3R_1 + R_3 & 3y - 8z = -27 \\ & & x + y + z = 9 \\ & -(3/2)\,R_2 + R_3 & 2y - 5z = -17 \\ & & -\tfrac{1}{2}z = -3/2 \end{array}$$

Shows that $z = 3, \quad y = -1 \quad and \quad x = 7.$

As these examples illustrate, Gauss' method uses the elementary reduction operations to set up back-substitution.

Definition: In each row, the first variable with a nonzero coefficient is the row's leading variable. A system is in echelon form if each leading variable is to the right of the leading variable in the row above it (except for the leading variable in the first row).

Example-4. The only operation needed in the examples above is pivoting. Here is a linear system that requires the operation of swapping equations. After the first pivot

$$\begin{array}{lll} x - y = 0 & & x - y = 0 \\ 2x - 2y + z + 2w = 4 & -2R_1 + R_2 & z + 2w = 4 \\ y + w = 0 & & y + w = 0 \\ 2z + w = 5 & & 2z + w = 5 \end{array}$$

The second equation has no leading y. To get one, we look lower down in the system for a row that has a leading y and swap it in.

$$R_2 \leftrightarrow R_3 \qquad \begin{array}{l} x - y = 0 \\ y + w = 0 \\ z + 2w = 4 \\ 2z + w = 5 \end{array}$$

(Had there been more than one row below the second with a leading y then we could have swapped in any one). The rest of Gauss' method goes as before.

$$-2R_3 + R_4 \qquad \begin{array}{l} x - y = 0 \\ y + w = 0 \\ z + 2w = 4 \\ \\ -3w = -3 \end{array}$$

Back-substitution gives $w = 1, \quad z = 2, \quad y = -1 \quad and \quad x = -1$

Strictly speaking, the operation of rescaling rows is not needed to solve linear systems. We have included it because we will use it later in this book as part of a variation on Gauss' method, the *Gauss-Jordan method*.

All of the systems seen so far have the same number of equations as unknowns. All of them have a solution and for all of them there is only one solution.

Example-5. Linear systems need not have the same number of equations as unknowns. This system

$$\begin{aligned} x + 3y &= 1 \\ 2x + y &= -3 \\ 2x + 2y &= -2 \end{aligned}$$

has more equations than variables. Gauss' method helps us understand this system also, since this

$$\begin{array}{rl} & x + 3y = 1 \\ -2R_1+R_2 & -5y = -5 \\ -2R_1+R_3 & -4y = -4 \end{array}$$

Shows that one of the equations is redundant. Echelon form

$$\begin{array}{rl} & x + 3y = 1 \\ -(4/5)R_2+R_3 & -5y = -5 \\ & 0 = 0 \end{array}$$

gives $y = 1$ *and* $x = -2$. *The* '$0 = 0$' is derived from the redundancy.

That example's system has more equations than variables. Gauss' method is also useful on systems with more variables than equations.

Another way that linear systems can differ from the examples shown earlier is that some linear systems do not have a unique solution. This can happen in two ways.

The first is that it can fail to have any solution at all.

Example-6. Contrast the system in the last example with this one.

$$\begin{array}{lcl} x + 3y = 1 & & x + 3y = 1 \\ 2x + y = -3 & -2R_1+R_2 & -5y = -5 \\ 2x + 2y = 0 & -2R_1+R_3 & -4y = -2 \end{array}$$

Here the system is inconsistent: no pair of numbers satisfies all of the equations simultaneously. Echelon form makes this inconsistency obvious.

$$\begin{array}{rl} & x + 3y = 1 \\ -(4/5)R_2 + R_3 & -5y = -5 \\ & 0 = 2 \end{array}$$

The solution set is empty.

Example-7. The prior system has more equations than unknowns, but that is not what causes the inconsistency – **Example-5** has more equations than unknowns and yet is consistent. Nor is having more equations than unknowns necessary for inconsistency, as is illustrated by this inconsistent system with the same number of equations as unknowns.

$$\begin{array}{lcl} x + 2y = 8 & \quad -2R_1 + R_2 \quad & x + 2y = 8 \\ 2x + 4y = 8 & & \quad 0 = -8 \end{array}$$

The other way that a linear system can fail to have a unique solution is to have many solutions.

Example-8. In this system

$$x + y = 4; \; 2x + 2y = 8$$

Any pair of numbers satisfying the first equation automatically satisfies the second. The solution set $\{(x, y) | x + y = 4\}$ is infinite – some of its members are (0,4),(-1,5) and (2.5, 1.5). the result of applying Gauss' method here contrasts with the prior example because we do not get a contradictory equation.

$$\begin{array}{lc} -2R_1 + R_2 \quad & x + y = 4 \\ & 0 = 0 \end{array}$$

Do not be fooled by the '0 = 0' equation in that example. It is not the signal that a system has many solutions.

Example-9. The absence of a '0=0' does not keep a system from having many different solutions. This system is in echelon form

$$x + y + z = 0$$

$$y + z = 0$$

has no '0=0' and yet has infinitely many solutions. (For instance, each of these is a solution: (0,1, -1), (0,1/2,-1/2), (0,0,0) and (0, -π,π). There are infinitely many solutions because any triple whose first component is 0 and whose second component is the negative of the third is a solution).

Nor does the presence of a '0=0' mean that the system must have many solutions. **Example-5** shows that. So does this system, which does not have many solutions – in fact it has none – despite that when it is brought to echelon form it has a '0 = 0' row.

$$\begin{array}{rcl} 2x - 2z = 6 & & 2x - 2z = 6 \\ y + z = 1 & \quad -R_1 + R_3 \quad & y + z = 1 \\ 2x + y - z = 7 & & y + z = 1 \\ 3y + 3z = 0 & & 3y + 3z = 0 \\ & & 2x - 2z = 6 \\ & -R_2 + R_3 & y + z = 1 \\ & -3R_2 + R_4 & 0 = 0 \\ & & 0 = -3 \end{array}$$

We will finish this subsection with a summary of what we have seen so far about Gauss' method.

Gauss' method uses the three row operations to set a system up for back substitution. If any step shows a contradictory equation then we can stop with the conclusion that the system has no solutions. If we reach echelon form without a contradictory equation and each variable is a leading variable in its row, then the system has a unique solution and we find it by back substitution. Finally, if we reach echelon form without a contradictory equation, and there is not a unique solution (at least one variable is not a leading variable) then the system has many solutions.

The next subsection deals with the third case – we will see how to describe the solution set of a system with many solutions.

Describing the solution set

A linear system with a unique solution has a solution set with one element. A linear system with no solution has a solution set that is empty. In these cases the solution set is easy to describe. Solution sets are a challenge to describe only when they contain many elements.

Example-10. This system has many solutions because in echelon form

$$\begin{array}{l} 2x + z = 3 \\ x - y - z = 1 \\ 3x - y = 4 \end{array} \quad \begin{array}{c} \\ -(1/2)R_1 + R_2 \\ -(3/2)R_1 + R_3 \end{array} \quad \begin{array}{l} 2x + z = 3 \\ -y - (3/2)z = -1/2 \\ -y - (3/2)z = -1/2 \end{array}$$

$$-R_2 + R_3 \quad \begin{array}{l} 2x + z = 3 \\ -y - (3/2)z = -1/2 \\ 0 = 0 \end{array}$$

Not all of the variables are leading variables. The Gauss' method theorem showed that a triple satisfies the first system if and only if it satisfies the third. Thus, the solution set $\{(x, y, z)|2x + z = 3 \; and \; x - y - z = 1 \; and \; 3x - y = 4\}$, can also be described as

$\{(x, y, z)|2x + z = 3 \; and \; -y - 3z/2 = -1/2\}$.

However, this second description is not much of an improvement. It has two equations instead of three, but it still involves some hard-to-understand interaction among the variables.

To get a description that is free of any such interaction, we take the variable that does not lead any equation, z, and use it to describe the variables that do lead, x and y. The second equation gives $y = (1/2) - (3/2)z$ and the first equation gives, $x = (3/2) - (1/2)z$. Thus, the solution set can be described as $\{(x, y, z) = ((3/2) - (1/2)z, (1/2) - (3/2)z, z)|z \in R\}$. For instance, (1/2, -5/2, 2) is a solution because taking $z = 2$ gives a first component of 1/2 and a second component of -5/2.

The advantage of this description over the ones above is that the only variable appearing, z, is unrestricted – it can be any real number.

Definition: The non-leading variables in an echelon-from linear system are free variables. In the echelon form system derived in the above example, x and y are leading variables and z is free.

Example-11. A linear system can end with more than one variable free. This row reduction

$$\begin{array}{l} x+y+z-w=1 \\ y-z+w=-1 \\ 3x+6z-6w=6 \\ -y+z-w=1 \end{array} \xrightarrow{-3R_1+R_3} \begin{array}{l} x+y+z-w=1 \\ y-z+w=-1 \\ -3y+3z-3w=3 \\ -y+z-w=1 \end{array} \xrightarrow[R_2+R_4]{3R_2+R_3} \begin{array}{l} x+y+z-w=1 \\ y-z+w=-1 \\ 0=0 \\ 0=0 \end{array}$$

ends with x and y leading and with both z and w free. To get the description that we prefer we will start at the bottom. We first express y in terms of the free variables z and w with $y=-1+z-w$. Next, moving up to the top equation, substituting for y in the first equation $x+(-1+z-w)+z-w=1$ and solving for x yields $x=2-2z+2w$.

Thus, the solution set is $\{2-2z+2w,-1+z-w,z,w)|z,w \in R)$.

We prefer this description because the only variables that appear, z and w, are unrestricted. This makes the job of deciding which four-tuples are system solutions into an easy one. For instance, taking $z=1$ and $w=2$ gives the solution (4, -2,1,2). In contrast, (3,-2,1,2) is not a solution, since the first component of any solution must be 2 minus twice the third component plus twice the fourth.

Example-12. After this reduction

$$\begin{array}{l} 2x-2y=0 \\ z+3w=2 \\ 3x-3y=0 \\ x-y+2z+6w=4 \end{array} \xrightarrow[-(1/2)R_1+R_4]{-(3/2)R_1+R_3} \begin{array}{l} 2x-2y=0 \\ z+3w=2 \\ 0=0 \\ 2z+6w=4 \end{array} \xrightarrow{-2R_2+R_4} \begin{array}{l} 2x-2y=0 \\ z+3w=2 \\ 0=0 \\ 0=0 \end{array}$$

x and z lead, y and w are free. The solution set is $\{(y,y,2-3w,w)|y,w \in R\}$. For instance, (1,1,2,0) satisfies the system – take $y=1$ and $w=0$. The four-tuple (1,0,5,4) is not a solution since its first coordinate does not equal its second.

We refer to a variable used to describe a family of solutions as a parameter and we say that the set above is parametrized with y and w. The terms ‘parameter’ and ‘free variable’ do not mean the same thing. Above, y and w are free because in the echelon form system they do not lead any row. They are parameters because they are used in the solution set description. We could have instead parametrized with y and z by rewriting the second equation as $w=2/3-(1/3)z$. In that case, the free variables are still y and w, but the parameters are y and z. Notice that we could not have parametrized with x and y, so there is sometimes a restriction on the choice of parameters. The terms ‘parameter’ and ‘free’ are related because, the solution set of a system can always be parametrized with the free variables. Consequently, we shall parametrized all of our descriptions in this way.

Example-13. This is another system with infinitely many solutions.

$$\begin{array}{rcl} x + 2y = 1 & & x + 2y = 1 \\ 2x + z = 2 & -2R_1 + R_2 & -4y + z = 0 \\ 3x + 2y + z - w = 4 & -3R_1 + R_3 & -4y + z - w = 1 \\ & & x + 2y = 1 \\ & -R_2 + R_3 & -4y + z = 0 \\ & & -w = 1 \end{array}$$

The leading variables are x, y and w. the variable z is free.

Notice here that, although there are infinitely many solutions, the value of one of the variables is fixed $w = -1$. Write w in terms of z with $w = -1 + 0z$. Then $y = (1/4)z$. To express x in terms of z, substitute for y into the first equation to get $x = 1 - (1/2)z$. The solution set is $\{(1 - (1/2)z, (1/4)z, z, -1) | z \in R\}$.

We finish this subsection by developing the notation for linear systems and their solution sets that we shall use in the rest of this paper.

Idea for Computational Program or

C-Program of Gaussian Elimination to solve Linear Equations

To solve for the stresses, we need to solve a set of equations with several unknowns. The number of unknowns increase as the number of elements and nodes in the truss increases. For a very complex truss, there would be many equations and unknowns.

We need a method that is very fast and can be used on a wide range of problems. Gaussian elimination meets these requirements. It has the following attributes:

a. It works for most reasonable problems
b. It is computationally very fast
c. It is not too difficult to program

The major drawback is that it can suffer from the accumulation of round off errors.

Upper Triangular Form (Forward Sweep)

We can illustrate how it works with the following set of equations.

$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 = d_1$

$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 = d_2$

$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 = d_3$

We would like to solve this set of equations for X_1, X_2, and X_3. We can rewrite the equations in matrix form as:

$$\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & d_1 \\ a_{21} & a_{22} & a_{23} & d_2 \\ a_{31} & a_{32} & a_{33} & d_3 \end{array}$$

We want to multiply the first equation by some factor so that when we subtract the second equations the a_{21} is eliminated.

We can do this by multiplying it by a_{21}/a_{11}. These yields:

$$\begin{array}{ccc|c} a_{11} & \left(\frac{a_{12}a_{21}}{a_{11}}\right) & \left(\frac{a_{13}a_{21}}{a_{11}}\right) & \left(\frac{a_{21}d_1}{a_{11}}\right) \\ a_{21} & a_{22} & a_{23} & d_2 \\ a_{31} & a_{32} & a_{33} & d_3 \end{array} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Subtracting the first equation in (1) from the second equation in (1) then replacing the first equation with its original form yields:

$$\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & d_1 \\ 0 & a_{22} - \frac{a_{12}a_{21}}{a_{11}} & a_{23} - \frac{a_{13}a_{21}}{a_{11}} & d_2 - \frac{a_{21}d_1}{a_{11}} \\ a_{31} & a_{32} & a_{33} & d_3 \end{array}$$

We do the same thing for the third row by multiplying the first row by a_{31}/a_{11} and subtracting it from the third row.

$$\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & d_1 \\ 0 & a_{22} - \frac{a_{12}a_{21}}{a_{11}} & a_{23} - \frac{a_{13}a_{21}}{a_{11}} & d_2 - \frac{a_{21}d_1}{a_{11}} \\ 0 & a_{32} - \frac{a_{13}a_{31}}{a_{11}} & a_{33} - \frac{a_{13}a_{31}}{a_{11}} & d_3 - \frac{a_{31}d_1}{a_{11}} \end{array}$$

We can reduce the complexity of the terms symbolically by substituting new variable names for the complex terms. This yields

$$\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & d_1 \\ 0 & b_{22} & b_{23} & e_2 \\ 0 & b_{32} & b_{33} & e_3 \end{array}$$

Now multiply the second row by b_{32}/b_{22} and subtracting it from the third row.

$$\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & d_1 \\ 0 & b_{22} & b_{23} & e_2 \\ 0 & 0 & b_{33} - \frac{b_{23}b_{32}}{b_{22}} & e_3 - \frac{b_{32}e_2}{b_{22}} \end{array}$$

Again we simplify by substituting in for the complex terms.

$$\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & d_1 \\ 0 & b_{22} & b_{23} & e_2 \\ 0 & 0 & c_{23} & f_3 \end{array}$$

This is what we call upper triangular form. We started at the first equation and worked our way to the last equation in what is called a forward sweep. The forward sweep results in a matrix with the value on the diagonal and above and zeroes below the diagonal.

Backward Sweep

We can solve each equation of the upper triangular form by moving through the list starting with the bottom equation and working our way up to the top.

$$X_3 = f_3/c_{33}$$

$$X_2 = (e_2-b_{22}X_3)/b_{22}$$

$$X_1 = (d_1-a_{12}X_2-a_{13}X_3)/a_{11}$$

This is called back substitution and the process as a whole is called the backward sweep.

Equation Normalization

You can see from the equations that it is very important for the diagonal coefficients to be non-zero. A zero valued diagonal term will lead to a divide by zero error.

In fact, there will be fewer problems with accumulated round off error if the diagonal coefficients have a larger magnitude than the off diagonal terms. The reason for this is that computers have limited precision. If the off diagonal terms are much larger than the diagonal term, a very large number will be created with the right-hand-side of the equation is divided by the diagonal term.

The size of this number dominates the precision of the computer and causing smaller values to be ignored when added to the larger value. For example, adding 1.3×10^{11} to 94.6 in a computer that has 8 digits of precision does results in a value 0f 1.3×10^{11}. The 94.6 is completely lost in the process. This can be seen in the previous example.

$$a_{22}- \frac{a_{12}a_{21}}{a_{11}}$$

If $a_{11}$is small compared to a_{22} then we could end up with

Small – large

In addition, loose the value of small (a_{22}) altogether.

This problem is solved by using two steps in the processing.

a. Normalize the rows so that the largest term in the equation is 1.
b. Swap the rows around so that the largest term in any equation occurs on the diagonal. This is called partial pivoting.

Diagonal dominance can usually be improved by moving the rows or the columns in the set of equations. Moving the columns is somewhat more difficult than moving the rows because the position of X_1, X_2, . . . changes when you move the columns. If you move the columns, you must keep track of the change in position of the Xs. When the rows are moved, the Xs stay in the same place so rows can be moved with the added complication.

Moving both rows and columns is called full pivoting and it is the best method for achieving diagonal dominance. We are going to look at partial pivoting because it works for many problems and is much simpler to implement. With partial pivoting, we are only going to move the rows.

Before we can perform either full or partial pivoting, we must normalize the rows.

Partial Pivoting

Now we can move the equations around (partial pivoting) so that the ones are on the diagonal. It is important to notice that normalization must be done first because without normalization it would be difficult to know how to rearrange the equation. There would be no basis for the row-by-row comparison.

Overall process

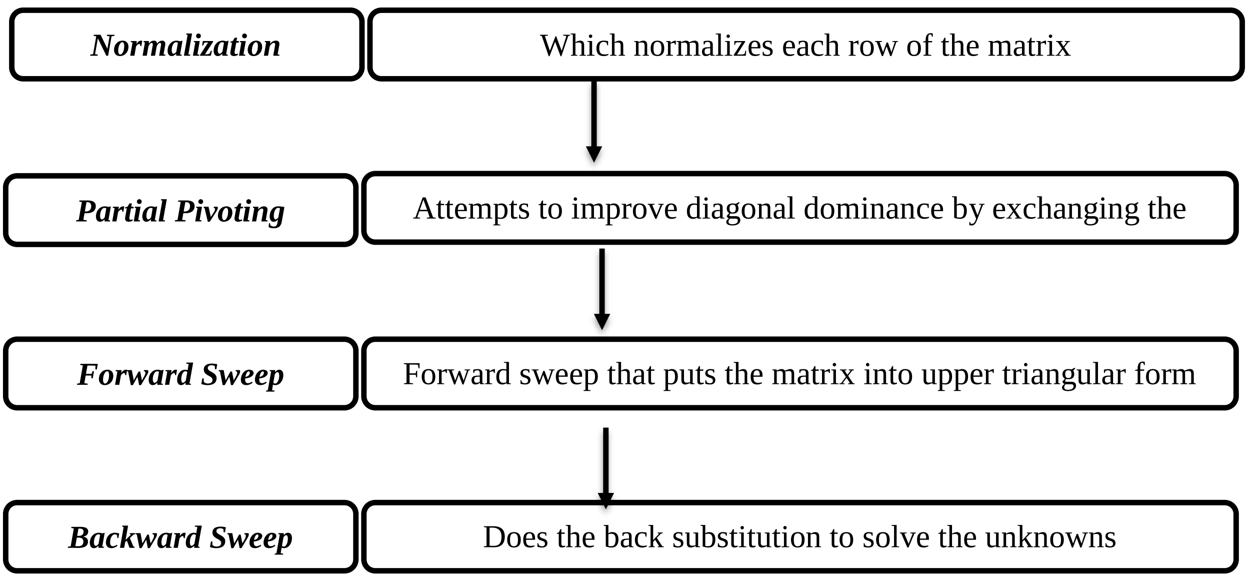
The overall process becomes

a. Normalize each equation
b. Move the equations to achieve diagonal dominance (partial pivoting)
c. Do a forward sweep that transforms the matrix to upper triangular form
d. Do a backwards sweep that solves for the values of the unknowns.

This algorithm can be written into very compact and efficient computer algorithms and is one of the more common ways of solving large numbers of simultaneous linear equations.

The following program reads a data file containing a matrix defining a set of linear simultaneous linear equations. The program then executes 4 functions to solve the system. The first function normalizes the equations, the second does partial pivoting to achieve diagonal dominance, the third puts the matrix in upper triangular form, and the forth function does the back substitution to solve the system of equation. The program is sized to handle up to 100 equations and 100 unknowns.

Four functions of Computational steps to solve the equations. They are:



These routines are very short and efficient.

CONCLUTION

This is how, this paper show the glimpse of idea about how to write the computational program code in various platforms by knowing the process of getting the solutions of system of linear equations. However using Gauss method by expressing those in terms of four step to solve the system of equations easily with accuracy and quickly. It is safe and quick never loses solutions or picks up extraneous solutions.

REFERENCES

1. **Kanneth Hoffman, Ray Kunze** -L*inear Algebra* :Second edition Prertice-Hall(1971)
2. **Donald E. Knuth** -*The art of computer programming* :Addison Wesley (1988)
3. **James R. Munkres** -*Elementary linear Algebra :*Addison-Wesley(1964)
4. **J. Komlos** -*On the determinant of (0.1) Matrice:* Studia Sci math Hungar-2 (1967).
5. **John R. Rice -***Numerical methods, Software and Analysis :* Second edition Academic press (1993.)
6. **Lary Smith** -*Linear Algebra :Second edition Springer* Verlag (1984)
7. **W.K. Nicholson** -*Elementary Linear Algebra* :PWS-Kent, Boston(1990)
8. **A.R. Magid** -*Applied Matrix models* :John wiley and sons, Newyork(1984)
9. **N.J. Pullman -***Matrix theory and its applications* :Marcel Dekker Inc. Newyork(1976)
10. **D. Hearn and M.P. Baker** -*Computer Graphics:*Scott, Forsman and company, Glenview, iIlinois (1988)